بسم ا الر ن الرحيم

3rd Islamic Astronomical Conference
Astronomical Applications in Islamic Shari'a
(Amman, Jordan; October 20th - 22nd, 2003)

Hijri Calendar & Lunar Visibility: Physical Approach

A. Haq Sultan
Faculty of Science, Physics Department, Sana'a University, Sana'a YEMEN

Abstract

Ancient civilizations relied upon the apparent motion of sun, moon, planets, and stars to determine days, months, seasons, and years. These bodies have provided us a reference for measuring time. Islamic calendar is based on lunar months, which should begin when the thin crescent Moon is actually sighted- for the first time- in the western sky after sunset after New Moon.

Many methods for predicting the Moon's appearance have been proposed throughout history and new models are still being developed. All these models have to be compared to published observations to test their validity.

We showed in this paper the necessity to a Unified Islamic Calendar. We tried also to develop a new model for predicting first lunar visibility depending on:

- Light intensity coming from the thin crescent and reaching the eye.
- Twilight brightness.
- New researches on human eye perception.

1. Introduction

The Islamic dates begin at sunset and end at the following day at sunset. It is the visibility of the crescent Moon that determines when the Islamic month begins. Weather conditions and differences in the observer's location explain why there are sometimes differences in Islamic dates. Moreover, in some existing Hijri calendars, Muharram 1, 1 A.H. corresponds to July 16, 622 AD, and in others it corresponds to July 15, 622 AD, The absence of a global criterion for first visibility still a challenge for researchers in astronomy and astrophysics.

Methods for predicting first sighting of the new crescent moon have been around since the time of the Sumerians (five thousand years ago). Efforts for obtaining an astronomical criterion for predicting the time of first lunar visibility go back to the Babylonian era (tow thousand years ago), with significant improvements and work done later by ancient Hebrew, Muslims scientists and others.

In the twentieth century Maunder (1911), Danjon (1932), Bruin (1977) Schaefer (1988), Ilyas (1994), McPartlan (1996), Yallop (1997), Fatoohi (1998) and Caldwell (2000) have developed empirical methods for predicting first sighting of the new crescent moon.

The factors to be considered for evaluating a physical method to predict a naked-eye first visibility of the lunar thin crescent are:
1) The geometry of the Sun, Moon, and horizon, 2) the width and surface brightness of the crescent, 3) the absorption of Moon light by atmosphere, 4) the scattering light in the atmosphere, and 5) the physiology and psychophysics of human vision.

We may summarize the above five factors to three physical quantities: The luminance of *the source* (The Moon) which depends on its astronomical position, light absorption of *the medium* between the body and the observer, and perception *the receiver i.e.* human vision Fig. 1.

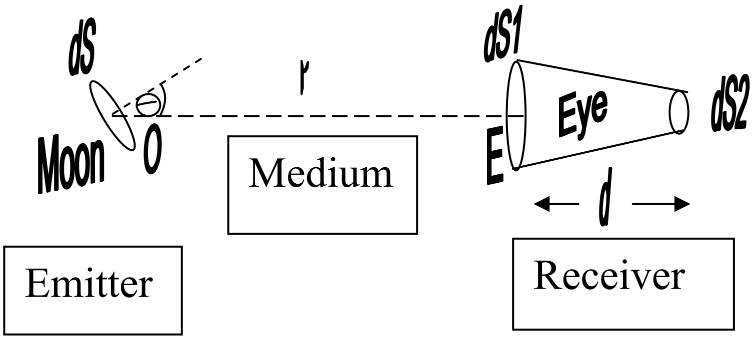

Fig. 1

In Fig. 1, let us consider the area *dS;* the fraction of illuminated disk of the Moon, as uniform diffusing object O, of luminance *L*, viewed by the eye at E. If the normal to *dS* makes an angle *Θ* to OE, the luminous intensity *I* in the direction OE is given by $I = L\ dS \cos\Theta$. This is the luminous flux per unit solid angle emitted by *dS*. Now the solid angle subtended at *dS* by the eye is dS_1/r^2, where OE = *r*. Thus the luminous flux *F* entering the eye = $L\ dS \cos\Theta * dS_1/r^2$ (Neglecting for the time being other intensities, from other objects, arriving to the eye and neglecting the reduction of signal due to atmosphere). If the retinal image has an area dS_2, the illumination of the image = luminous flux per unit area of image = $(L\ dS \cos\Theta * dS_1)/(r^2 * dS_2)$.

But dS_2 is the image of the object *dS* in the eye-lens. Thus if the distance from E to the retina is ***d*** we have $(dS \cos\Theta / dS_2) = (r^2 / d^2)$.

Hence, Illumination of image on retina = $L\ dS_1 / d^2$. But dS_1 / d^2 is a constant. Consequently the image on retina depends only on luminance *L.*

The retina is the beginning of the nervous system that transports information to the brain. It contains neural circuitry that converts light energy to action potentials that travel out the optic nerve into the brain. A direct 3-neuron chain is the basic unit of transmission (Photoreceptor to bipolar cell to ganglion cell). Light falls on retina, transformed to action potential that ganglion cells convey to the brain.

Retina is not like a video camera. It does not send a point by point intensity of its illuminations. Rather the important feature the retina detects is contrast, i.e. differences in light intensity between points in the retina.

One way to express the luminance contrast C is as follows:

$$C = (L - L_B)/ L_B$$

Where L = surface brightness of the stimulus (moon)

L_B = surface brightness of the background (Twilight sky)

Contrast Threshold (C_{th}) is a measure of the ability of an observer to distinguish a minimum differences in surface brightness between two areas a given percentage of the time.

Size is the most generally recognized and accepted factor in seeing. Contrast threshold is dependent upon the size of an object, which affects the size of the image on the retina. The important aspect of size is not the physical size of the object, but the visual angle that the object subtends at the retina. Under good lighting conditions with good eyesight a person can discern 0.15 milliradian or 0.5 minute of arc.

According to Ricco's law, which states that, within a critical diameter, the product of image area and light intensity (quanta/area) is constant, Ganglion cells breaks the visual world into a large number of small circular receptive fields Fig.2.

But the visibility of stimulus is not simply a matter of its luminous intensity; it also affected by its contrast with the background illumination (Blackwell's data). From Ricco's law and Blackwell, we may consider the thin crescent as a group of disks of varying angular size, each has its Equivalent Blackwell Disk. The biggest disk is at the center of the crescent. Disks become smaller when we go in the direction of the cusps. Their thresholds become bigger, at certain moment C_{th} becomes >C, that is why we don't see the whole crescent during early observations.

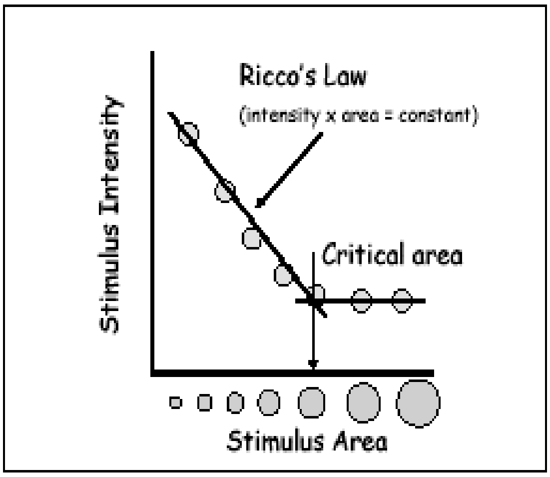

Fig.2

2. Methode

2.1 The source

To evaluate the luminance L_* of the thin crescent moon at the moment of observation, we used the following equations:

- The total visual magnitude (m) of the moon is given by

$$m = -12.73 + 0.026\, a + 4*10^{-9} a^4, \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (1)$$

whre a is the phase angle in degrees (1)

- The actual (extra-atmospheric) surface brightness of the moon (L_*) in tenth magnitude stars per square degree (S_{10}) is given by

$$L_* = (1/D)\, 2.51^{(10-m)} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (2)$$

where D is the illuminated area of the moon surface (2).

2.2 The medium

The western sky twilight brightness (L_B) was measured at Al-Wadi, 20 Km north of Sana'a (2000 m height, 44º 12′ E, 15º 24′ N). The measurements achieved on 2002 December 22, using a PHYWE selenium photocell (45-mm diameter, corrected to human eye; calibration with metal filament lamp at 2850º K). Data are represented in table I.

Table I

The twilight brightness (L_B) was measured of Al-Wadi

Sun altitude in degrees	0	-0.5	-1	-1.5	-2	-2.5	-3	-3.5	-4	-4.5	-5	-5.5	-6	-6.5	-7
Sky luminance in nL	7.3E7	4.7E7	3.1E7	2E7	1.2E7	7.5E6	4.5E6	2.6E6	1.5E6	8.5E5	4.8E5	2.7E5	1.5E5	8.7E4	5E4

Then we calculate the effect atmospheric using the following equations:

- The apparent (ground-observed) surface brightness L of the moon in S_{10} du to atmospheric will be

$$L = L_* e^{-kX} \quad (2) \quad \ldots\ldots\ldots \quad (3)$$

where k is the atmospheric extinction given by Walker (1987) (3)

and X is the air mass calculated by using Rozenberg equation (quoted by Krisciuans and Schaefer) (4)

$$X= (\cos(Z)+0.25e^{-11\cos(Z)})^{-1} \quad \ldots\ldots\ldots \quad (4)$$

Z is the zenith distance of the moon in degrees

- The apparent surface brightness L in nanolamberts (nL) (4)

$$L = 0.263L_* e^{-KF} \quad \ldots\ldots\ldots \quad (5)$$

2.3 The receiver

- Contrast C between L and twilight sky brightness (L_B) will be

$$C = (L - L_B)/ L_B \quad \ldots\ldots\ldots \quad (6)$$

- And finally, the visibility criterion is given by

$$C >= C_{th} \quad \ldots\ldots\ldots \quad (7)$$

where C_{th} is Blackwell's contrast threshold

3.Calculations & discussions

3.1 Concept of the best time

For the first visibility of the thin crescent moon, we have to know when is the best time for making the observation i.e. when the contrast between the crescent moon and the twilight sky is becoming sufficient for the Moon to be seen.

Schaefer (1988) calculated the best time from the logarithm of the actual total brightness of the Moon divided by the total brightness of the Moon needed for visibility for the given observing conditions. Based on Bruin (1977), Yallop (1998)[5] found a simple formula for "best time" in minutes after sunset, which is equal to 4/9 * the lag time of the moon.

Using our model, we found that "best time " is dependent on the elevation of observation site. When the site is with moderate elevation (less than 1000m), the crescent may be seen shortly after sunset (Stamm with elevation of 860m, spotted the crescent moon just 7 minutes after sunset on 1996 January 20. On the contrary to Stamm, Patchick (with elevation of 1740m) spotted the same crescent on the same day, but 28 minutes after sunset[6].

We will see later that Al-Mostafa has spotted the moon only 5 minutes after sunset.

3.2Abbreviations used in table II

Ph.angle: phase angle in degrees, equation (1).
Semi-dia: Semi-diameter of the lunar disk.
Mag: lunar magnitude (m), equation (1).
%Ill: illuminated fraction of the lunar disk.
L_*: extra-atmospheric luminance of the moon (nL), equation (2).
L: ground-observed luminance of the moon (nL), equations (3) and (5).
W: Topocentric width of the lunar disk in minutes of arc.
L_B: Twilight sky luminance (nL).
C: Contrast between L and L_B
C_{th}: Blackwell Contrast Threshold C_{th}
V: Visibility.

3.3Assumptions and applications adopted in table II

* We consider the perfect geometrical situation with the difference in azimuth between the Sun and the Moon at the moment of observation equals zero i.e. DAZ = 0.

* In equations (3) and (4), we applyAl-Wadi elevation with its best time of observation i.e. Moon altitude =2 degrees.
* We used the local L_B, measured in the previous section.

Table II

Visibility of the crescent moon of March 2002

Date UT	h	m	Ph.angle o	Semi-dia '	Mag	Elon o	% ill	L_* (nl)	L (nl)	W '	L_B (nl)	C_{th}	C	Vis
14 3 2002	2	5	175.004	14.683	-4.43	4	0.19	4.3E8	2.4E7	0.06	1.2E7	60	1	no
14 3 2002	7	5	174.513	14.684	-4.48	4.6	0.23	3.7E8	2.1E7	0.07	7.5E6	43	2	no
14 3 2002	9	20	174.021	14.684	-4.54	5	0.27	3.4E8	1.9E7	0.08	4.5E6	41	3	no
14 3 2002	11	20	173.482	14.685	-4.6	5.6	0.32	3E8	1.7E7	0.1	2.6E6	38	6	no
14 3 2002	12	50	173.027	14.686	-4.65	6.1	0.37	2.7E8	1.5E7	0.11	1.5E6	33	9	no
14 3 2002	14	20	172.539	14.686	-4.7	6.6	0.42	2.5E8	1.4E7	0.12	8.5E5	31	15	no
14 3 2002	15	50	172.023	14.687	-4.75	7.1	0.48	2.3E8	1.3E7	0.14	4.8E5	32	26	no
14 3 2002	17	16	171.508	14.688	-4.81	7.6	0.55	2.1E8	1.2E7	0.16	2.7E5	37	43	yes
14 3 2002	18	40	170.989	14.689	-4.86	8.1	0.62	2E8	1.1E7	0.18	1.5E5	39	72	yes
14 3 2002	19	57	170.501	14.690	-4.92	8.6	0.68	1.9E8	1.1E7	0.2	8.7E4	43	125	yes
14 3 2002	21	12	170.017	14.691	-4.97	9.1	0.76	1.8E8	1E7	0.22	5E4	46	199	yes

From the above table, we may conclude that elongation of 7.6 degrees is the lowest naked eye visibility limit, with site elevation around 2000m. Fatoohi et al[7] examined Danjon limit using a large number of ancient and modern observation of first and last visibility. Fatoohi got empirically same results of our physical method.
Can we reach Danjon limit (7 degrees) with our naked eye observation?
Our answer yes, with altitude around 3000m,and perfect geometrical situation DAZ = 0.

3.4 Testing the model with the latest documented observation

We have tested our method with the observation of Zaki A. Al-Mostafa published in (*The Observatory*) p. 47, February 2003.

On 2002 March 14, in Laban (46.45 degree E, 24.6 degrees N), using a telescope, Al-Mostafa spotted the crescent five minutes after sunset i.e. at 15:07 UT.

Explanations concerning calculations in table III:
Row A = Naked eye observation with local conditions of Laban, we used Schaefer's algorithm[8] to evaluate L_B.
Row B = Considering our method assumptions (Site elevation = 2000 m, DAZ=0, moon altitude = 2degrees, and Al-Wadi L_B.)
Row C = The crescent could be seen from Laban if we use a telescope or binoculars with magnification >10

Table III

Al-Mostafa observation

Date UT	h	m	Ph.angle o	Semi-dia '	Mag	Elon o	% ill	L_* (nl)	L (nl)	W '	L_B (nl)	C_{th}	C	Vis

A) 14 3 2002	15	7	172.27	14.687	-4.73	7	0.45	2E8	9.8E6	0.13	1.4E6	26	6	no
B)									1.3E7	0.13	4.8E5	40	26	no
C)												0.35	6	yes

4.Conclusions

1- At site elevation of 2200 m, the crescent could be seen, if it's elongation greater than or equal to 7.6 degrees.

2- Our method confirms the 7.5 degrees empirically studied by Fattohi et al.

3- The method explains length shortening of the (new moon).

4- The method shows that the "best time" is directly proportional to site elevation.

5- The method predicts that, naked eye observation may reach Danjon limit.

6- We propose to consider Muharram 1, 1 A.H. starts at the sunset of July 15, 622 AD.

7- According to our model, the first day of Hijri month may starts at Makah sunset, if the geocentric elongation exceeds 7.5 degrees at the moment of sunset in Makah.

References

1- Schaefer B. E, PASP, 103,645-660, July 1991.

2- Sidney O. Kastener, JRASC, Vol. 70, No.4, 1976.

3- Garstang R. H, PASP, 103, 1109-1116, October 1991

4- Krisciunas K., Schaefer B. E, PASP, 103,1033-1039, September 1991.

5- Yallop, B.D. *RGO NAO*, Tech. Note 69. 1997

6- Pepin M. B. *Sky & Telescope*, 104-106, December 1996.

7- Fatoohi, L.J., Stephenson, F.R., Al-Dargazelli, S.S., (1998), *The Observatory*, 118, 65–72.

8- Schaefer B. E, *Sky & Telescope*, 57, May 1998.