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EXPLAINING AND CALCULATING THE LENGTH OF THE NEW CRESCENT MOON

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Danjon noticed that the length (cusp to cusp) of the new crescent moon was less than 180 degrees and suggested that the cause of the shortening is the shadows of the lunar mountains. McNally, however, attributed the crescent shortening to atmospheric seeing, while Schaefer suggests that length shortening is due to sharp falling off of the brightness towards the cusps.

We attribute length shortening to the Blackwell contrast threshold; we consider the thin crescent as a group of discs of varying angular size, and each has its equivalent Blackwell disc, the largest being at the centre of the crescent. The discs become *smaller* in the direction of the cusps, therefore the Blackwell thresholds become *higher*. According to this model, if we know the apparent diameter of the Moon and the width of the crescent, we can calculate the approximate visible length of the crescent.

Introduction

In 1931 August, Danjon[1] at the Strasbourg Observatory, France, noticed that the length (cusp to cusp) of the crescent Moon, which was only 16.6 hours before conjunction, extended only 70–80 degrees instead of 180 degrees. He suggested that the cause of the shortening is the shadows of the lunar mountains. However, McNally[2] did not accept this interpretation and showed that the height of the mountainous lunar terrain compared to the lunar radius is not sufficient to be the cause of shortening; he attributed the crescent shortening to atmospheric seeing. Schaefer[3], in turn, rejected McNally's explanation, showing that the resolution of the human eye is larger than the size of the crescent disc so that seeing has no effect on the perceived width. He suggested that length shortening is due to sharp falling off of the brightness towards the cusps, emphasizing that detection threshold does not depend on the surface brightness of the Moon but on the total brightness integrated across the crescent. He points out[3] that as the extreme parts of the crescent are narrower than the resolution of the eye "the detection threshold does not depend on the surface brightness of the Moon, but on the total brightness integrated across the crescent".

Discussion

We disagree with Schaefer because we believe that human vision is more sophisticated than he suggests. Briefly, human vision involves the simultaneous interaction of the eye and the brain through a network of neurons, receptors, and other specialized cells. The retina contains neural circuitry that converts light energy to action potentials that travel along the optic nerve to the brain. If the retinal area that is illuminated is small enough then the photons will fall entirely within the centre of the receptive field. If enough photons fall into the receptive field, the ganglion cell will respond by firing. According to Ricco's Law of spatial summation, if we increase the area of the stimulus so that it is still within the centre of the receptive field, then more photons would be collected over this larger area and so a lower intensity of light would be required. Ricco's Law of spatial summation has been completely disregarded by Schaefer: “the experiments (see also Blackwell 1946) show that unresolved circles of light have a threshold that is independent of the source size, yet which depends on the total brightness within the circle. The basic idea is that all that matters is how much light is received by any resolution element of the eye and not how the light is spread over the ‘pixel’.”

As far as I know Schaefer is the most prolific contributor to the subject that we are discussing, but his paper[4] implies a measure of confusion with some photometric definitions such as *brightness*, *surface brightness*, *integrated brightness*, and *total integrated brightness.* In spite of this (*i.e.*, if we neglect Schaefer’s length-shortening interpretation), he got very good empirical, results represented by his Fig. 2.

Clark describes Blackwell’s 1946[4] data in his excellent book[5] entitled *Visual Astronomy of the Deep Sky*; Clark has added additional comments since the book's publication (1990), at http://clarkvision.com/visastro/omva1/index.html. At this site, Clark illustrates Blackwell’s 1946 data in a diagram (his Fig. 2.6) which he explained as follows: “Here we notice that for objects with small angular sizes, the smallest detectable contrast times the surface area is a constant. As an object becomes larger, this product is no longer constant. The angle at which the change occurs is called the critical visual angle. An object, smaller than this angle, is a point source as far as the eye is concerned. (A point can be considered the angular size smaller than which no detail can be seen.)”

In effect, Clark specifies the domain of utilization of Ricco's Law, which should be less than the critical visual angle. For stimuli smaller than the critical visual angle, *i.e.*, on the left side of Clark's figure, we conclude two things: the first was mentioned by Clark himself while the second was mentioned neither by Schaefer nor by Clark. (*i*) When the contrast between surface brightness of the stimulus and surface brightness of the background is higher than Blackwell's contrast threshold, the object is seen as a point source; and (*ii*) when it is less than Blackwell's threshold, the object couldn't be seen at all. Therefore, the visibility of small stimuli — smaller than the critical visual angle — is characterized by the left-side curves of Clark's Figure. From these curves, we find that the stimulus visibility always depends on its luminance, its diameter, and the background luminance even if it is smaller than the critical visual angle.

Crescent-length calculations

We consider the thin lunar crescent as a group of discs of varying angular size, and each has its equivalent Blackwell disc. The largest disc is at the centre of the crescent. Discs become smaller in the direction of the cusps, therefore Blackwell thresholds become higher. To obtain Blackwell's contrast threshold, C_{th}, for discs of diameters less than 0.6 minute of arc, we extrapolate the data in table VIII of Blackwell[4]. Fig.1 is an example of our extrapolations. We found that the smallest width to be seen depends mainly on three local factors: site elevation, sky luminance, and the zenith distance of the Moon at the moment of observation. It is also depends on whether the Moon is near apogee or perigee. For our site at Mouneef (1990 metres, 44° E, 13° N), the two smallest widths to be seen are about 0.16 arc minute when the Moon is near apogee and about 0.18 arc minute when the Moon is near perigee.

From standard software and Fig. 2, we can calculate the length of the visible crescent as follows: on the vertical diameter of the Moon's disc (Fig. 2), the length of the group of discs starting with the disc at the centre and ending at the end of one cusp is equal to $r + W/2$, then

$$r + W/2 = W + W_1 + W_2 + \ldots$$

$$= W + W\,[1 - W/(r + W/2)] + W\,[1 - W/(\mathrm{r} + W/2)]^2 + \ldots \qquad (1)$$

From Danco[5], the right side of Eqn.1 represents a decreasing geometric progression whose sum is equal to $W/W/(r + W/2)$, which is clearly equal to the left side of Eqn.1.

In Fig. 2, let $L/2$ be the distance starting at the centre of the Moon's disc and ending at the beginning of the largest invisible width w (for our site w = 0.14 arc minute when the Moon is near apogee and w = 0.16 arc minute when the Moon is near perigee)

$$L/2 = r - w/W/(r + W/2)$$

$$= r - w\,(r + W/2)/W$$

then,

$$L = 2r - 2w\,(r + W/2)/W,$$

where L represents the vertical component of the visible crescent length in arc minutes. Then the visible crescent length in degrees will be: $L/2r \times 180°$.

If we take w = 0.15 arc minute as an approximate value for the biggest invisible width, we get a simple formula for calculating crescent length. It depends only on two factors: the apparent diameter of the Moon, D, and the width of the crescent, W. Then

$$L = D - 0.3\,(D + W)/2W. \qquad (2)$$

Putting the model to the test

In the following, we test our model calculations using two well-documented observations. We ask the reader to compare our results with those given by the only existing model — Schaefer's model.

Danjon's observation of 1932 August 13: (reported length = 70–80°)

D = 32.88 arc minute

W = 0.27 arc minute

$L = D - 0.3\,(D + W)/2W$

= 14.46 arc minutes

The crescent length in degrees will be

$$L/D \times 180° = 79.2°$$

By using MOONC60 software (http://www.starlight.demon.co.uk/mooncalc), which adopts Schaefer's model, we get $L/D \times 180° = 93°$

Stamm's observation of 1996 January 20: (reported length[6] = 45°)

D = 33.32 arc minute

W = 0.19 arc minute

$L = D - 0.3\,(D + W)/2W$

= 6.9 arc minute

The crescent length in degrees will be

$$L/D \times 180° = 37.1°$$

By using Schaefer's model we get $L/D \times 180° = 56°$

Conclusions

Our model gives a new explanation for the cause of length shortening of the new crescent Moon and it introduces a method for calculating the approximate visible length of the thin crescent Moon. Further, our photometric model gives the same results obtained by Schaefer's empirical model but with a different interpretation concerning the cause of length shortening.

References

(1) A. Danjon, *L'Astronomie*, **46**, 57, 1932.

(2) D. McNally, *QJRAS*, **24**, 417, 1983.

(3) B. E. Schaefer, *QJRAS*, **32**, 265, 1991.

(4) H. R. Blackwell, *JOSA*, **36**, 624, 1946.

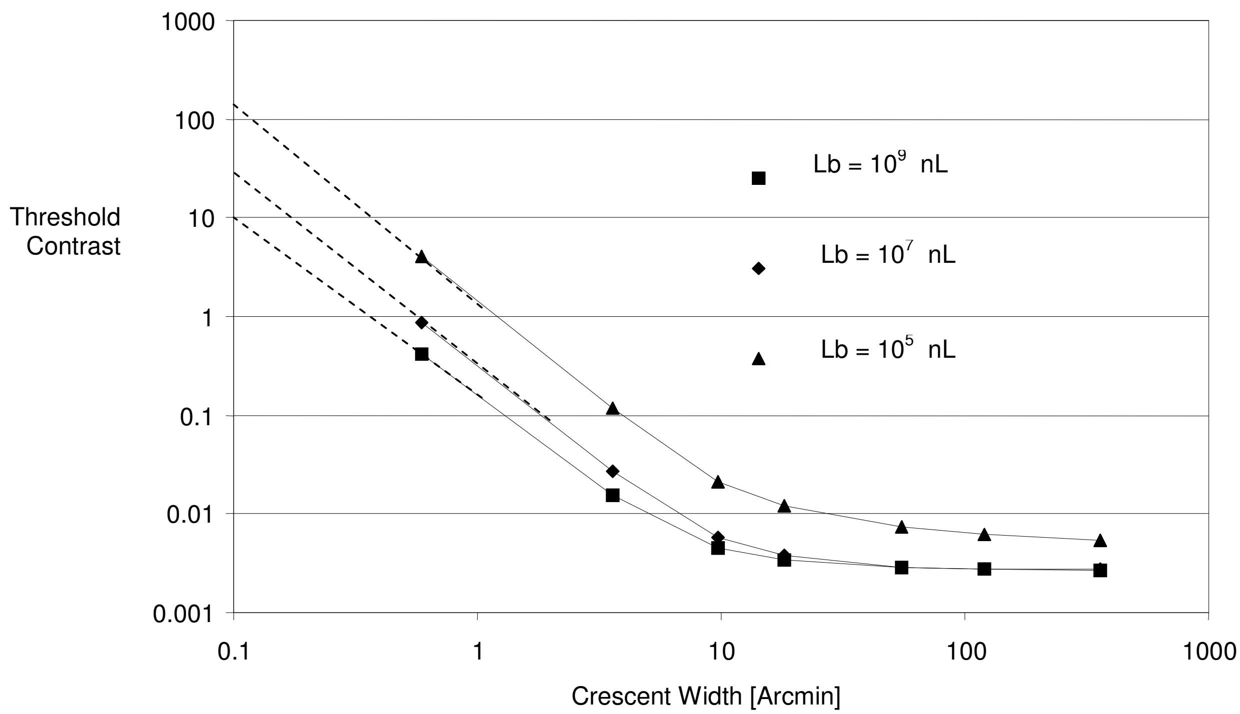
(5) P. E. Danco, *Higher Mathematics in Problems and Exercises* (Mir Publishers,Moscow), 1983.

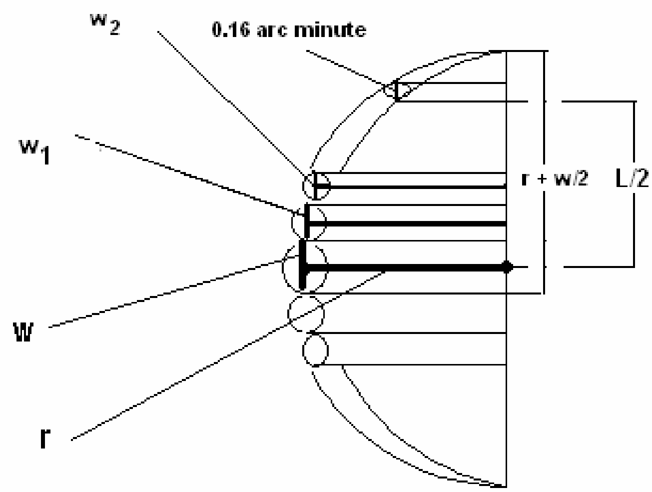
(6) M. B. Pepin, *S&T,* **92**, 104, 1996.

Figure Captions

FIG. 1: An example of our extrapolation of the data in table VIII of Blackwell[4], the crescent width is equal to 0.1 arc minute.

FIG. 2. Starting from the centre of the crescent and going in the direction of the cusps, the crescent contains two groups of discs; each group represents a decreasing geometric progression. The visible crescent length in degrees will be: $L/2r \times 180°$, where L represents the vertical component of the visible crescent length in arc minutes and is given by $L = 2r - 2w\,(r+W/2)/W$, and where r is the apparent semi-diameter of the Moon, W is the width of the crescent, and w is the largest invisible width. For our site $w = 0.14$ arc minute when the Moon is near apogee and $w = 0.16$ arc minute when the Moon is near perigee.


1000
100
10
1
0.1
0.01
0.001
Threshold
Contrast
Lb = 10^9 nL
Lb = 10^7 nL
Lb = 10^5 nL
0.1
1
10
100
1000
Crescent Width [Arcmin]


w2
0.16 arc minute
w1
r + w/2
L/2
w
r