Orientation Specification For Mosques

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ABSTRACT:
This paper puts a framework for translating religious (Sharia) requirements for turning towards Mecca into geometrical requirements. Another objective of this paper is to investigate the sources of error in finding the direction, and to suggest acceptable limits for the error given the current state of technology without causing undue difficulty. In the process we differentiate between error for individuals and for mosques, and in this work we focus on mosques. Initially, we examined individual perception of direction error. We found it very subjective and would be hard to produce a suitable error limit. Next we used the condition to stay within Mecca as another possible approach to find the limits. Finally, factoring implementation issues, we gave our recommendations for the specs.

INTRODUCTION:
Turning towards Mecca (Qibla direction) is a prerequisite for a Muslim prayer. While some time the correct direction is not clear, due diligence in finding the Qibla is required. Muslims have turned in the early days of Islam towards Jerusalem, then, they were ordered to turn towards Mecca. Finding the right direction is not a trivial task, it involves the use of coordinate system, and the knowledge of spherical geometry. Such tools enable us to solve the problem easily nowadays, but in the past, people have to resort to different ways to find the Qibla, chief among them is the use of astronomical alignments. These days, it is almost agreed upon that the correct direction is by following the great circle path, however, it is not explained in terms of the requirement of the prayer, and instead it is advocated because it is the shortest distance between the observer and the target (Mecca). That explanation has given ammunition to the proponents of Rhumb line (the straight line between the two points on the Mercator projection or map) as the more direct line to the target, and they argue that the shortest distance is not part of the religious requirements. Therefore, there is a need for stating the requirements clearly and translating that into geometrical specifications to help in settling endless debates about the subject.

REQUIREMENTS:
The requirement for turning towards Mecca is stated in the Quran as:

فول وجهك شطر المسجد الحرام و حيث ما كنتم فولوا وجوهكم شطره

"Turn your face towards the sacred Masjid, and wherever you were turn towards it" (2,144)

Clearly, the requirement can be fulfilled by facing the Kabba, however, if the person doesn't see the Kabba, then we have to assume that a tower is extended over the Kabba and goes up vertically until it can be seen by our observer. Therefore the requirement becomes facing this imaginary tower if we don't see the Kabba itself.

GEOMETRICAL EXPRESSION OF THE REQUIREMENT:
Before we get into translating the requirement into geometrical conditions, we would like to introduce some terms:
Azimuth angle: The angle measured from the North in clockwise direction. Sometimes the terms: Bearing[1], and Orientation are used for the same purpose.
When a person faces a target, that person and the target form a plane as shown in Figure 1. The direction of the plane with the North depicts the azimuth angle or the bearing of the target. Therefore, to face the Kabba, the observer and the Kabba have to be in the same plane, and we call this plane "the observation plane".

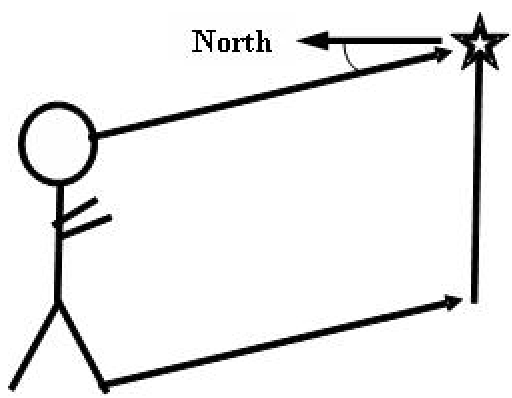

Figure 1. The plane of observation that contains the observer and the target

Such an arrangement would meet the requirement as stated in the Quran, and that is true if we were on a flat surface or on the surface of a sphere.
If the observer and the target are on the surface of a sphere, as in the case of being on the surface of the earth, then we still have a plane that combines the observer and the target. The plane cuts a circle into the sphere. This circle is called the great circle as can be seen in Figure 2. The observer is at point A, and rises to H, while the target is at B and extends to T.

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[1] See Appendix

The vector AH, and the vector BT intersect at point C (the center of the sphere as well as the center of the great circle) and form a plane -the observation plane- such a plane intersects with the sphere producing the great circle. The chord AB is the direct straight line between the observer at A and the target at B, while the arc AB is the most direct route on the surface of the sphere. Note that the plane of the great circle (which is the same as the observation plane) contains the points A,H,B,T, and C, and therefore, the chord AB as well as the line of sight HT belong to the great circle plane too.

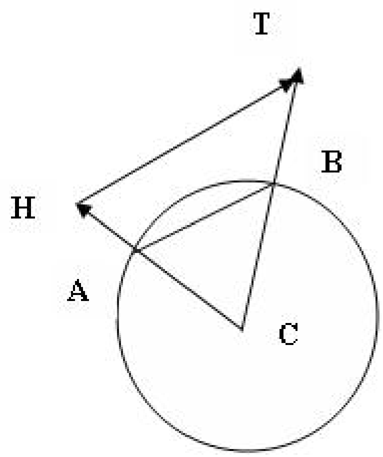

Figure 2. The Great Circle plane containing the Observer AH, and Target BT

The plane would make an angle with the North, and that would give the azimuth angle. To calculate the azimuth angle, we use spherical geometry relations. The needed information is the coordinates (latitude and longitude) of the target, -in this case it is Mecca- and the current location –observer location. The formula for calculating the azimuth angle is:

$$\theta = ATAN2\left[\frac{\sin(b1-b2)*\cos(a1)}{\sin(a1)*\cos(a2)-\cos(b1-b2)*\cos(a1)*\sin(a2)}\right]$$

Where:
θ: The azimuth angle (radian).
ATAN2: Trigonometric Inverse tangent function with two arguments.
Sin: Trigonometric Sine function.
Cos: Trigonometric Cosine function.
a1, b1: The latitude and longitude respectively of Mecca.
a2, b2: The latitude and longitude respectively of current location.

There are other ways to find the azimuth angle, but this is the most straightforward.

FINDING CARDINAL DIRECTION:

After calculating the azimuth angle for the Qibla, we need to find the North direction or any other cardinal direction. To do so, the first thing people will think about is the use of magnetic compass as a stand alone instrument or as part of mobile devices such as "Smart Phone". While it is straightforward to use this device, it is not recommended for finding Mosque orientation because it could introduce large errors. More accurate methods should be used for such a purpose. One possible method is the use of the Sun. It is well known that the shadow of objects in the northern hemisphere points to the true north at the transit time (local noon). The other method is to use ubiquitous satellite maps that give the direction for the north.

Once the north is identified, the direction of the Qibla is found by plotting the calculated azimuth angle.

As people try to turn to the right direction, there is going to be errors that would be part of the process, and in the following sections we will try to analyze them.

ERRORS DUE TO THE SHAPE OF THE EARTH:

The earth is nearly a sphere with the equatorial radius a = 6378.1370 km, while the polar radius b = 6356.7523 km. The average radius of the sphere is calculated in different ways, for example, according to the International Union of Geodesy and Geophysics (IUGG):

$$R = \frac{2a + b}{3}$$

R = 6371.009 km (3958.761 mi = 3440.069 nmi).

To increase accuracy for navigation, the oblate earth is modeled as an ellipsoid of revolution. An example for such a model is the World Geodetic System WGS 84 that is used in GPS systems. In this model, the equatorial section is a circle, while the polar section is an ellipse that has major axis as equatorial diameter, and minor axis as the polar diameter. Such an ellipse would have eccentricity e = 0.08182, and compression ratio of k = 0.99665. A cross section of this model is shown in Figure 3. In the figure the difference between the circle and the ellipse is exaggerated, otherwise, it will be very difficult to distinguish the two shapes.

For the purpose of finding the Qibla orientation, the error due to the use of a spherical model is very small, for example, the difference between the spherical model and WGS-84 when calculating the Qibla orientation from Detroit is about 0.07° or 4 arc-minutes, and that is 0.13 %.

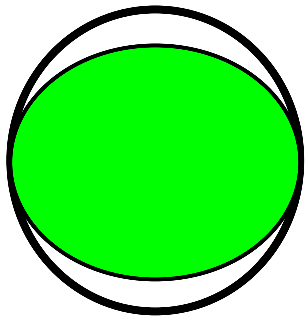

Figure 3. Exaggerated ellipse and circle representing earth oblateness

Therefore, modeling earth as a perfect sphere is a very good model.

ERRORS IN DIRECTION DUE TO STANDING IN A STRAIGHT LINE:

Another potential source of error stems from the arrangement of prayer in lines. In this case, if the imam is in the middle and in the direction of the Kabaa, the people in the line (straight line) will not be in the direction of the Kabaa.

Obviously, inside the Sacred Masjid, people pray in arcs so that they face the Kabaa. But once we go outside the Sacred Masjid, it becomes very quickly unnecessary to pray in an arc formation, and straight line becomes the practical manifestation of turning towards Kabaa as depicted in figure 4. It is easy to show that the error due to straight line formation is very small with the following example.

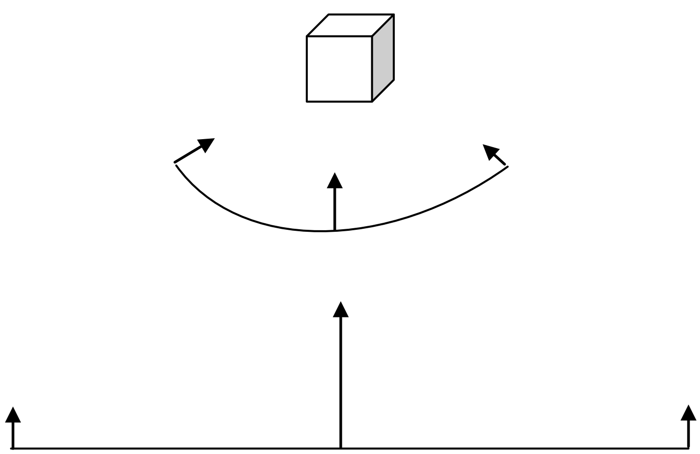

Figure 4. Prayer Line Formations

The height of Kabaa is 13.1 m, so on a perfect spherical earth with no mountains or valleys, theoretically, it should be visible from around 13 Km. If we have a straight prayer line with a width of 400 meters (would have about 1200 persons) and the Imam in the center is in the exact direction of Kabaa, then, the person on the edge would have the most error. If the person on the edge would turn towards the Kabba, then, his angle shown in Figure 5 would be given by the equation:

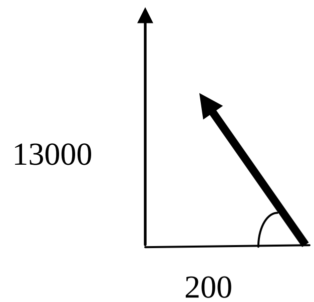

Figure 5. Example of the angle correction

$$\alpha = ATAN(\frac{13000}{200}) = 89.1^\circ$$

Therefore, for the person standing in a straight line at the edge of this very long line he would have an error of less than one degree.
Of course the above scenario is fictional, and the Kabaa is in a valley, and therefore, it wouldn't be possible to view it from such a distance. In addition, normal lines are much shorter than that. Therefore, if we combine normal line width with a longer distance from Kabaa, then, the error quickly diminishes, and it becomes clear that such an error is not an issue.

For example, the direct distance between the Masjid of the Prophet in Medina and Kabaa is about 340 Km, and therefore, for a prayer line with a width of 400 meters, the error for the person on the edge would be 0.03 degrees.
Even inside Mecca, for a distance of 5 Km from Kabaa, and for a line of 50 meters (about 150 persons in such a line), the error for the person on the edge of the line would be slightly less than 0.3 degrees.
Therefore, it is not an issue even for people very close to Kabaa let alone people who are far away, in addition, such an error will not affect the Imam of the prayer, therefore, it is not an issue.

OTHER ERRORS:
There are errors in the coordinates due to GPS errors for example, however these errors depend on the technology used, but overall they are really small, for example, Garmin (www8.garmin.com/aboutGPS/) advertised its newer models as having an error of less than three meters on average, and this will produce very insignificant error.
Some people have pointed to the plate tectonics as a source of error. In very simple terms, the earth crust consists of huge plates that move with respect to each other, therefore, locations can be constantly changing. The continents are moving a few centimeters a year, and at such a rate, it would take about a hundred thousand years for the error to start getting noticed.

HOW TO SET A STANDARDS FOR ORIENTATION ERROR:

How accurately should we turn towards Kabaa ?

It is a legitimate question that requires some analysis, but before we get into that, we suggest that we should distinguish between two things: orientation for Masajid, and orientation for individuals.

This differentiation is necessary because the Masjid is a fixed structure that is used by many, many people, and besides being a symbol for Muslims, the Masjid is used as a reference, therefore special care should be taken in deciding its orientation.

Some people have argued that there should be no specification because they want to maintain "the simplicity" and straightforwardness of the faith. However, this argument in our opinion has no merit. For one thing, mosques these days are structures that cost millions of dollars, and have to meet local codes, and are designed by engineers, and not by ordinary average folks. It makes no sense to pay attention to the smallest details, but when it comes to the primary purpose of the facility –prayer and its requirements- poor and casual work is acceptable ? !.

As for the individual, due diligence is required, but circumstances for individuals are different, therefore, different outcomes are normal, and it would be hard to demand a specific accuracy, however, it is useful to examine how people perceive error in orientation.

PERCEPTION OF ERROR IN ORIENTATION:

To find a limit for acceptable error, we set to examine the perception of orientation and when the deviation from a set direction is noticed.

For that purpose, we have conducted a small experiment. The experiment was done as follows:

A volunteer would stand facing a set direction as well as randomly sequenced deviations from that direction by 2, 3, 4, 5, 6, and 7 degrees to the right and to the left.

The students (viewers) would have to mark on their forms one of four choices:

1- The volunteer is straight (no deviation or Neutral)

2- Maybe (not sure there might be a deviation)

3- Slight deviation

4- Clear deviation.

The number of participating students was 25.

Results:

The collected data showed wide scatter all over the place, indicating that individuals are really different in their perception of angular deviation. So in order to get meaningful result, we have tried several schemes to represent the data, and we found that aggregating the incidents of detection gave us the clearest picture. In this scheme, we combined the categories: clearly, slightly, and half the incidents of not sure. This gave us the number of incidents of detection.

The graph in Figure 6 below shows the angle of deviation v.s. the percentage of detection incidents. So the x-axes shows the angular deviation of the volunteer from the set direction or the neutral. For example 4L is four degrees to the left, while the y-axes represents the percentage of detection incidents, or the percentage of people who detected a deviation in the direction.

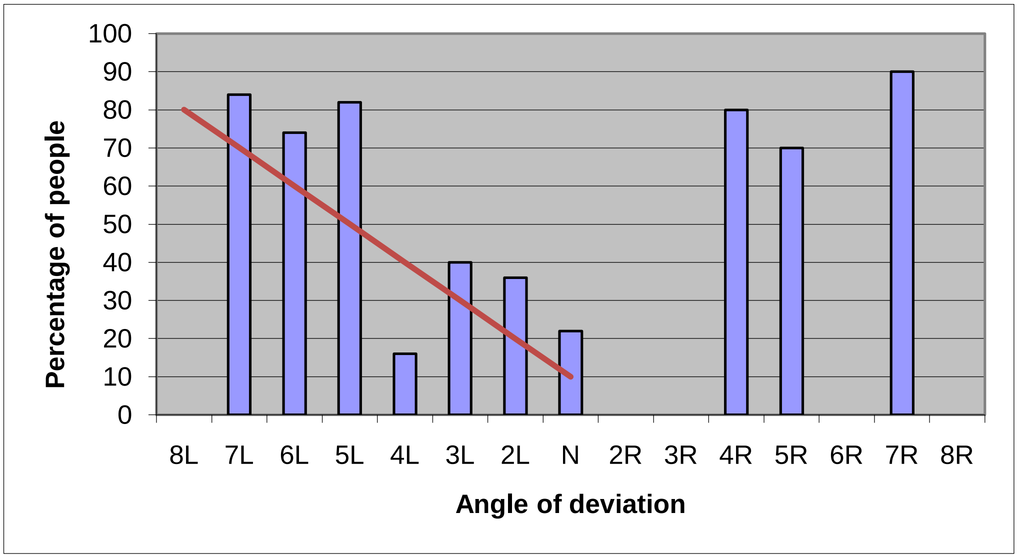

Figure 6. The percentage of people who perceive deviation as a function of angle of deviation

Keeping in mind the statistical nature of the data, a number of observations can be made:

1- The perception of deviation grows as the angle of deviation increases, and that makes sense. The increase in the number of people who notice the deviation as the deviation grows can be modeled in a number of ways, however, we chose to approximate it by a linear relation (the red line).
2- The perception of deviation appears to be symmetric.
3- The results for 4L (4 degrees left) can be explained by the fact that this was the first set the students evaluated.
4- The majority of people recognize the no deviation (Neutral).
5- If we use the straight line approximation for the data, then over two third of the people would recognize the deviation once it exceeds 7 degrees.

Of course, more experiments are recommended to draw more concrete conclusions, however, people perception of the error can't be used to set a standard because it varies greatly from one person to the other, and if we go with say seven degrees as an acceptable error, we will have a good number of people who would see the error to be excessive, so we look at another approach.

DISTANCE INDUCED AS A RESULT OF ERROR IN ORIENTATION:

One way to look at the effects of the error in the bearing of the Masjid is to find the resulting deviation at the destination point. Ideally, we would like to stay within the city of Mecca if possible.

To ease the analysis, we choose the direct straight line between the two points, and how this line changes as the azimuth angle changes.

The arrangement is explained in Figure 7.

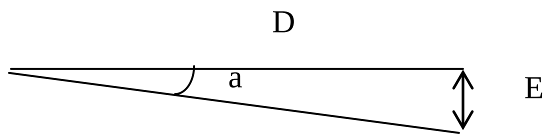

Figure 7. The effect of the error in the azimuth angle on the destination

Where:

D: the direct straight line distance between the origin point (observer) and destination.

E: The error from the destination point.

a: The error in the azimuth angle.

The relation between the three quantities is given by:

$$\tan(a) = \frac{E}{D}$$

However, if we know the direct distance "L" on the sphere, we can use it instead of D, where:

$$D = 2R\sin(\frac{L}{2R})$$

Where:

R: Radius of the earth (6371 Km).

For small azimuth error, we can use the following equation:

$$E = 2aR\sin(\frac{L}{2R})$$

The maximum error occurs when we are close to being on the other side of the globe from our destination, say, at D = 12000 Km, then, for an error of 1° in the azimuth angle, the error E = 222 Km ≈ 139 miles.

The Miqat are areas where people start their Hajj ritual (cloth change) is given in figure 8. The shape of the area is longer along the north-east axis. However, if we overlaid a circle that contains good part of this area, we can use such an area to identify the errer area. The circle has a radius of 59 miles, so roughly speaking the maximum error allowed (for a distance of D = 12000 km) if we want to stay within the circle is ± 0.45° degrees.

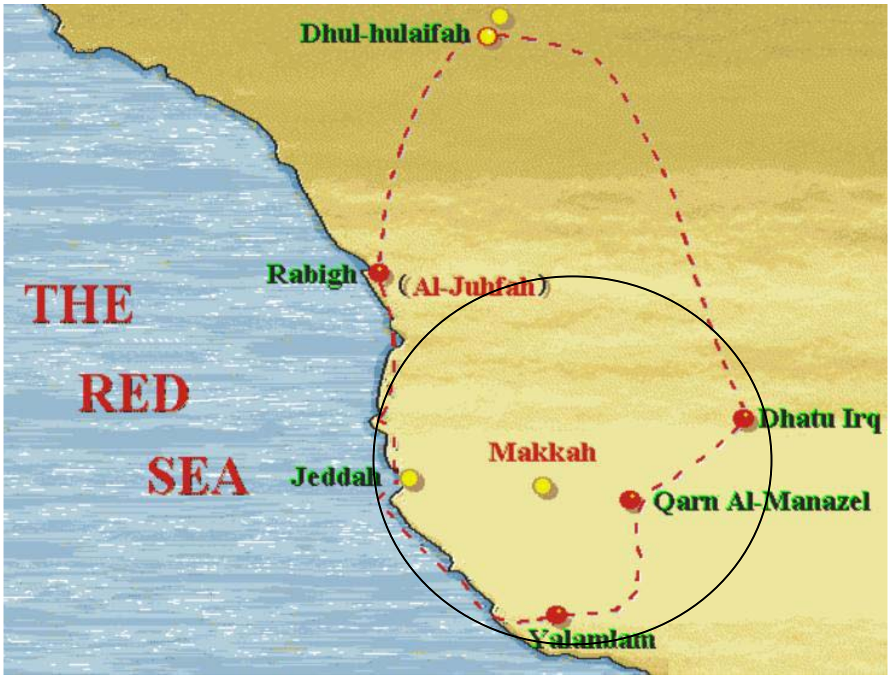

Figure 8. Miqat map for Hajj

<u>RECOMMENDATIONS AND IMPLEMENTATION ISSUES:</u>

The two major sources of error for finding Qibla are the error of approximating the shape of the earth by a sphere. Such error is on the order of less than 0.1 degrees. The other source is in charting the cardinal direction (like the North direction). Such an error is the dominant error and it is an implementation issue, - a function of the measuring instrument.

In the previous section we found out that the maximum orientation error in the worst case scenario should be less than ± 0.50° degrees. However, for most of the globe, the error could be larger and we can still be within our circle.

For example, for Detroit, L = 10800 Km, and D = 9531 Km, an error of ± 0.50° will put us within a radius of 50 miles, well within our circle.

In practice, an error of ± 0.50° degrees can be achieved easily using simple instruments, so such a limit is practical and easily implementable without any undue difficulty.

CONCLUSION:

In this work, we have translated the religious requirement for turning towards Mecca into a geometric condition: ***Both the observer and the target should be in the same plane***, and as a result, we can state that the direction given by the great circle is the correct direction for the Qibla. Then, we examined the sources of error that can occur when setting up the Qibla, and those errors come primarily from the error in identifying the cardinal directions, and to a smaller extent from approximating the shape of the earth as a perfect sphere.

In this work, we suggest that there should be a distinction between the error specification for houses of worship and individuals.

Then, the perception of people of orientation error was examined experimentally, and it was found that once the error exceeds seven degrees, the majority of people start noticing the error. However, the results reflect great difference between people, therefore, this approach can't be used for giving suitable specs.

A different approach was used. It is based on analyzing the allowable error in the orientation if we wanted to roughly stay within the Miqat region around Mecca. We approximated the region by a circle of a radius of 54 miles.

Meeting this requirement depends on the location on earth (the distance from Mecca). So if we have the longest possible distance, the allowable error should be less than ± 0.50° degrees, and we think this is a good and easily implementable spec.

Therefore, the error for the houses of worship should not exceed ± 0.50° degree, as this accuracy could be easily achieved with very simple tools, and not introduce any undue difficulty for people responsible for orienting mosques.

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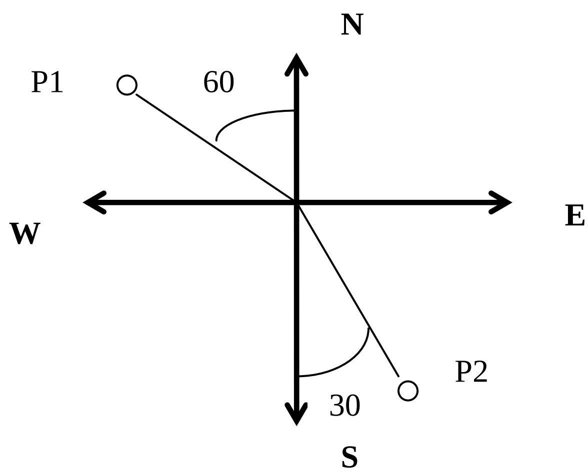
APPENDIX:

The term Bearing is used in navigation and surveying as follows:
Bearing is the acute angle between the line of sight of the point and the N-S direction.
Example:



The point P1 has a bearing of N 60° W, while the point P2 has a bearing of S 30° E