The Extended Crescent Visibility Criterion

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Abstract: **Crescent visibility has been a concern for determining the start of any lunar month. Various criteria have been offered by the astronomers since the Babylonians. The indigenous criterion proposed in this paper uses the two reliable parameters, altitude and crescent width, and makes it possible to estimate the visibility for any phase of the Moon, not just limited to thin crescents. Though very simple, the algorithm presented here produces rather consistent results. Various visibility graphs are included. In addition is introduced a tool for demonstration.**

Keywords: ***crescent visibility; Moon illuminance; sky brightness; crescent width; Sun-Moon elongation***

1. INTRODUCTION

The first visibility of the waxing crescent has always been a matter of interest for many societies. The word "month" has the same root as the word "Moon" and, in a lunar calendar, a month is defined as the time slice between two maiden appearances of the youngest crescent. A month for example in the Islamic calendar begins on the day following the first evening during which the waxing crescent becomes visible. Thus, for the preparation of a lunar calendar in advance, it is necessary to constitute valid formulae for the computational determination when a crescent may become visible. Astronomers therefore have strived to express various lunar visibility criteria since the Babylonian age.

This paper will introduce an alternative criterion for the naked-eye visibility of the lunar crescent. To aid the comprehension of the case, the physical perception mechanisms for the Moon's visibility will be presented first. Historical background about visibility criteria will then be explained briefly. After expressing the methodology of the new simplified criterion, the application developed for the demonstration of this criterion will be explained. Consecutively, the results obtained by this tool and their comparisons with other criteria will be summarized.

We start out by elaborating the generic lunar visibility problem.

2. PERCEPTION OF THE CRESCENT

For any object with sufficient size to be visible in the sky, there must be sufficient contrast between the object and the surrounding background [1]. Contrast is defined as the ratio of the object's (Moon's in this case) illumination to the sky's brightness [2]. So the brightness of the Moon must be a certain level higher than the sky brightness at that azimuth and elevation.

The angle between the Sun-Moon and Earth-Moon lines is called elongation. The Moon phase angle is defined as the projection of this angle onto the ecliptic plane, i.e. the difference between the celestial longitudes of the Earth and the Moon. At the time of conjunction when the celestial longitudes are the same, the Moon phase angle will be zero and elongation becomes a minimum. This minimum elongation will be an angle (Moon declination angle at conjunction) bearing a value between +5.15° and -5.15°, since Moon's orbital plane is tilted at 5.15° with respect to the ecliptic (see Figure 1). A solar eclipse occurs if this elongation is smaller than the Moon parallax, approx. 1°.

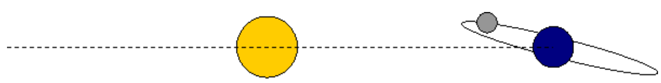

Figure 1 – Moon Declination

While the Moon in its gibbous phase is also visible during daytime, a thin crescent can only be seen after sunset, since the sky is so bright before the sunset that a new crescent is impossible to detect. As the Sun depresses further below the horizon, the sky brightness uniformly decreases. The perceived brightness of the illuminated portion of the Moon (crescent) depends upon his elongation; the sky brightness, on the other hand, is mainly related to the position of the Sun. This fact implies that the lower the Sun moves, the more will be the contrast between the thin crescent and the twilit sky. Nevertheless, the waxing crescent will also set soon after the Sun. The time lag between sunset and moonset depends on the Sun-Moon elongation and the latitude, as depicted in Figure 2 When the Moon approaches the horizon, adverse effects like atmospheric refraction as well as clouds, fog, dust or pollution will diminish the brightness of the Moon and deteriorate the contrast [3].

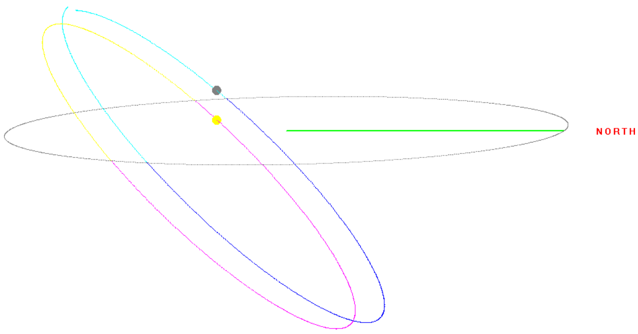

Figure 2 – Sun/Moon Trajectory

A thick crescent, lagging a sufficient amount behind the Sun, can be distinguished above the horizon during a certain period after sunset until it vanishes within the last few degrees of elevation. The younger the crescent is, the later can it be detected and the earlier it will disappear. There will be a limiting condition, where a crescent can just be identified for a very few minutes. This boundary is called the first (earliest) visibility of a crescent. Such a crescent is also perceived as shorter than full 180°, as discovered by Danjon, because the thinner edges will fall below the physiological visibility threshold [12][20]. The most favorable instant for the visibility is denoted as the "best time" and the least elongation for a crescent to become visible is expressed by the "Danjon limit". There will also be a unique place on Earth for each lunation, where the crescent can be first observable globally. The Sun/Moon trajectory and position at the best time and place, calculated according to the novel criteria proposed in this paper, are displayed in Figure 2. Methods for the determination of the best time and the coordinates of the best place will be presented later.

3. PREVIOUS WORK

As stated in the previous section, the crescent must be brighter than the sky in order to be visible by the observer. This implies that any visibility criterion has to manifest at least two parameters; one for the crescent illumination, the other for the sky brightness [4]. Nevertheless, in some cases (especially in the ancient times), also single parameter approaches have been practiced.

We shall summarize the basic parameters found in the literature, as follows:

3.1 Lag

Lag, which is expressed as the time delay in minutes between the sunset and the moonset, is one of the oldest parameters, used since the Babylonian era. As a fact, the more the lag, the bigger will be the elongation. A greater elongation in turn leads to a thicker crescent, implying higher illumination. On the other hand, a bigger lag means that the Sun goes deeper below horizon before the Moon vanishes, resulting in a darker sky. In general, the required contrast will depend upon the lag. However, the contrast cannot be determined by the lag only; the illumination as well as the sky brightness is related to other parameters also, especially the latitude. In high latitudes, the Sun & Moon trajectories become more decumbent, which means that the lag increases for the same elongation.

3.2 Age

Age, the other simple parameter, is defined as the time in hours passed since the conjunction. Age is only a moderate indication of the crescent illumination, since it considers neither the speed / distance of the Moon, nor the declination angle. It hardly ever gives any information about sky brightness.

3.3 Altitude

The altitude difference between the Sun and the Moon is a more recent parameter. It is also known as Arc of Vision (ARCV). For a specific point on the sky (azimuth / elevation), the brightness gradually decreases as the Sun goes down. There is also a brightness gradient on the sky in vertical direction for a specific time, i.e. as one looks downwards from the zenith to the horizon, the brightness will grow with increasing zenith angle. Hence we can deduce that the brightness of the sky at the elevation of the Moon is a direct function of the ARCV. Therefore the altitude is a very good parameter to represent the sky brightness. In Figure 3, the western sky brightness is depicted as a function of the solar depression angle [11].

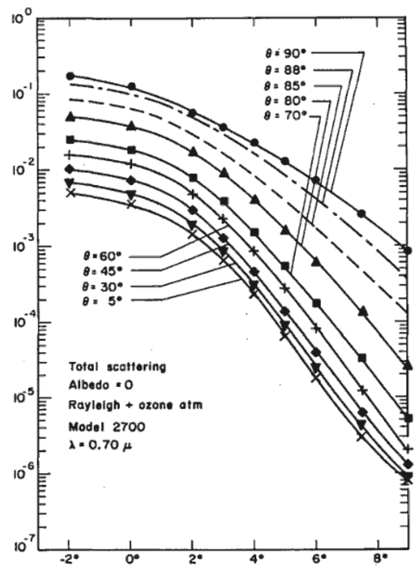

Figure 3 – Sky Brightness vs. Sun Altitude

Alternative parameters representing the altitude can be:

- Moon altitude at sunset
- Moon altitude when the Sun is 4° below horizon, which is regarded as nearly the best time [8].
- Apparent altitude of the crescent's lower limb

3.4 Azimuth

Sun-Moon azimuth difference is a common parameter generally used together with the altitude. This represents the crescent illumination. It is commonly abbreviated as DAZ (Delta Azimuth). In fact, DAZ and ARCV constitute the two orthogonal angles (see Figure 4) and, using the spherical trigonometry, one can write [16]:

```
cos(ARCL) = cos(ARCV) * cos(DAZ)
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ARCL stands for Arc of Light, which is anonymous to Sun-Moon separation, or elongation.

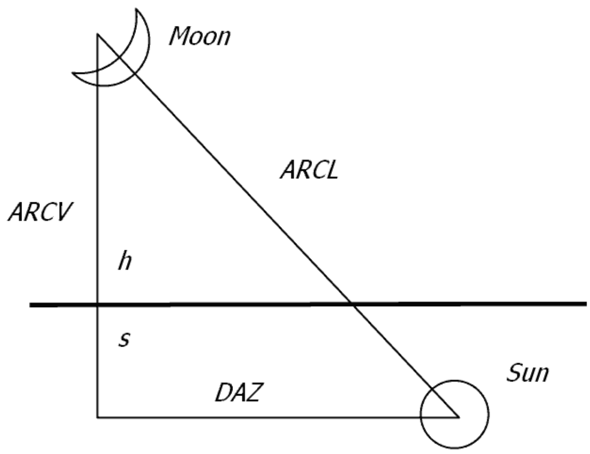

Figure 4 – Relation between ARCV, DAZ and ARCL

Azimuth and altitude together are used as two common parameters by most recent astronomers. The visibility criteria are generally shown in a graph (Figure 5). Using the above formula, we should note that for a given ARCL (elongation), ARCV (altitude) will increase as DAZ decreases. When DAZ = 0, ARCL will be maximum and equal to ARCV. The maximal Moon altitude for a specific elongation corresponds to minimal sky brightness for a given Moon illumination, maximizing the contrast. Hence the latitude where the azimuth becomes zero (the Moon is directly above the Sun) will be the unique place with the best visibility (see Figure 2).

The y-axis of the graph in Figure 5 (where DAZ = 0 and ARCL = ARCV) shows the minimum possible elongation for naked-eye visibility. Except for Fotheringham, this angle has a value of 10-11° based on statistical observation data.

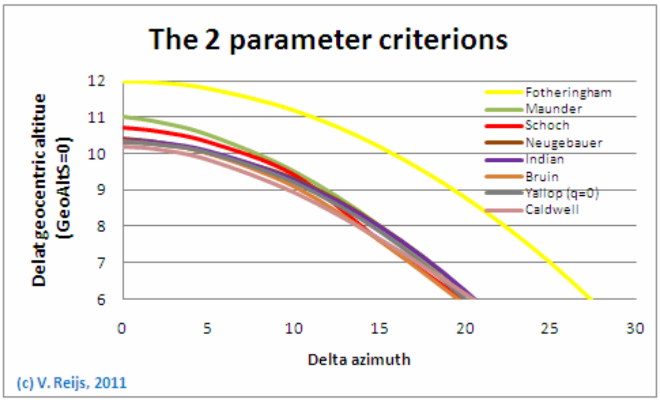

Figure 5 – Azimuth/Altitude Criteria

Ilyas extended this curve [9] in 1988 for large azimuth differences (high latitudes), as shown in Figure 6, which is known as Ilyas (C) criterion.

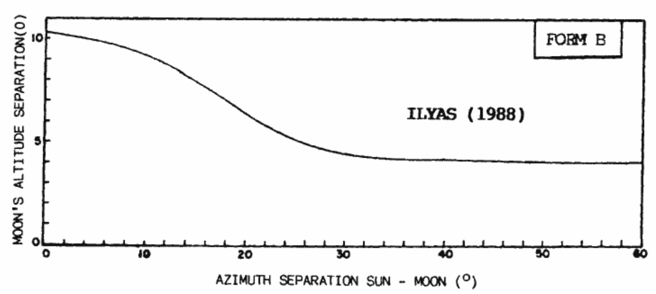

Figure 6 – Composite Extended Criterion of Ilyas

3.5 Elongation

Elongation has a strong relationship with the Moon illumination: Mathematically, the illuminated portion of the disc is given by the formula:

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Illumination = ½ * [1 - cos(Elongation)]
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For small angles, the illumination is proportional to the square of the elongation. As can be seen from the following equation [15], the elongation takes the declination angle into consideration:

```
cos(Elongation) = cos(Phase) * cos(Declination)
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Ilyas, in 1984, plotted a curve with Moon's altitude vs. elongation [10], denoted as Ilyas (A). The Royal Greenwich Observatory (RGO) also uses altitude & elongation. Caldwell recently presented a paper explaining the dependence of crescent visibility on lag & elongation [1]. In Indonesia, lag, elongation, altitude and azimuth are used in combination [17].

3.6 Crescent Width

Although elongation is a direct representation for the illumination of a disc, it lacks the size. Since the distance of the Moon to the Earth is not constant due to the eccentricity of the Moon's orbit, its apparent diameter changes continually. The central width of the crescent is directly proportional to the illuminated area observed and therefore should be incorporated to optimally represent the illumination criterion. The crescent width subtends an angle small enough to write the following approximation:

```
Width ≈ 11950 * Illumination / Moon Distance
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In this formula, the crescent width is in arc-minutes and the Earth-Moon distance is in thousands of kilometers.

Bruin developed a theoretical graph in 1977, plotting the Moon altitude and ARCV versus Sun depression using different values of crescent width, as shown in Figure 7. For example with a crescent width of 0.25' and ARCV of 10°, the crescent will remain visible as long as the Sun is between 2~8° of depression (points A and B). Schaefer adopted this model incorporating atmospheric correction factors [13].

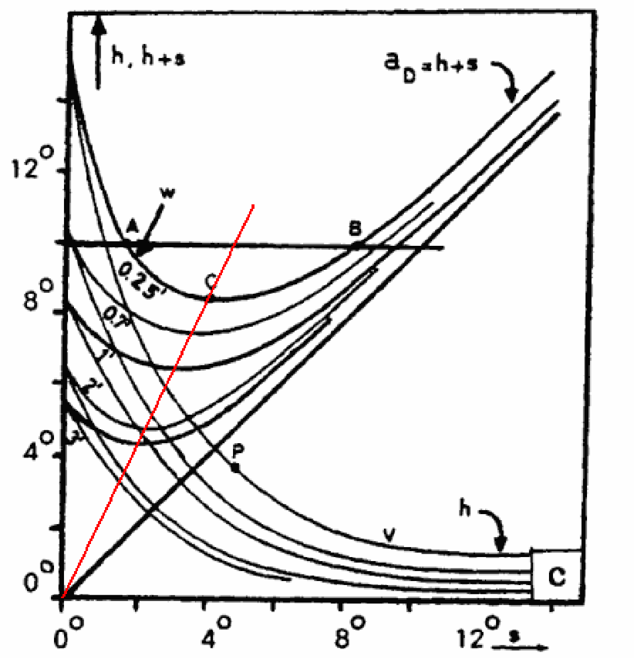

Figure 7 – Bruin's Criterion

Yallop, based on 295 observations, offered in 1997 a 3rd degree polynomial using ARCV and crescent width (W), where q is the visibility factor [15]:

$$q = ARCV - 11.8371 + 6.3226*W - .7319*W^2 + .1018*W^3$$

Visibility factor q is divided into 4 zones, ranging from "visible with naked eye" to "not visible even with optical aid". This criterion produces a curve with the known shape as in Figure 5; ARCV decreases with increasing DAZ (growing W).

Odeh, after investigating 737 records, proposed in 2006 to modify the offset value of the Yallop's equation [4].

Similarly, Qureshi attempted in 2005 to fit polynomials of 3rd degree for the available observation records [6].

3.7 Ease of Visibility in General

Criteria with two parameters investigated so far, namely the one for crescent illumination and the other for sky brightness, has been formulated by Hoffman by combining them into an ease-of-visibility parameter *v*, where *d* represents the sky darkness and *b* the Moon's illumination:

$$v = d + k * b$$

He then defined a normalized ease-of-visibility parameter *q*. Values < 0 will mean "crescent impossible to see" and values > 1 will have a meaning of "certainly visible" [2]. Variables v_0 and v_1 are the ease-of-visibility values for the lower and upper limit, respectively:

$$q = (v - v_0) / (v_1 - v_0)$$

He next asserted that the Moon will be visible if the following condition is met:

$$h_s < x * q + h_{s0} \pm \sigma h_{s0}$$

The parameter h_s is the instantaneous Sun altitude, h_{s0} the Sun altitude at the lower visibility limit and *x* an empirically fitted constant.

Note that while the criteria mentioned before describe the conditions for the earliest visibility of a crescent, Hoffman's rule should be valid for a broad range of altitudes, suitable for the aim of our work.

After elaborating the parameters and their pro-cons, we deduced that the most two suitable parameters would be the crescent width (W) for the crescent illumination and the altitude (ARCV) for the sky darkness. Hoffman applied in his work the square-root of the crescent width. Besides he used DALT instead of ARCV, which is the topocentric altitude difference. For small angles, DALT = ARCV - 1. We will denote the Sun altitude as S instead of h_s and the topocentric Moon altitude as M. So we may write:

$$v = DALT + k * \sqrt{W}$$
$$v = M - S + k * \sqrt{W}$$
$$S < x*(M-S + k*\sqrt{W} - v_0)/(v_1-v_0) + h_{s0} \pm \sigma h_{s0}$$

Assuming that:

$$y = x/(v_1-v_0)$$
$$z = y/(1+y)$$

The inequalities become:

$$y*M - (1+y)*S + y*k*\sqrt{W} > y*v_0 - h_{S0} \pm \sigma h_{S0}$$
$$z*M - S + z*k*\sqrt{W} > z*v_0 - (h_{S0} \pm \sigma h_{S0})/(1+y)$$

Hoffman suggested following values to be used with DALT and √W:

k	= 6.4	v_0	= 11.3
x	= 2.2	v_1	= 16.3
h_{S0}	= -6.1	σh_{S0}	= 1.43

Hence we calculate y = 0.44 and z = 0.31. Thus we may write:

$$0.31*M - S + 1.96*\sqrt{W} > 7.69 \pm 0.99$$

Following deductions can be made from this inequality:

- For a given Moon altitude, the limiting Sun altitude should increase (more sky brightness) as the crescent width increases (more illumination); in order the visibility (contrast) to remain the same.
- For a given Sun altitude, the limiting Moon altitude should decrease (more sky brightness) as the crescent width increases (more illumination); in order the contrast to remain the same.
- For a fixed crescent width, the limiting Moon altitude should decrease (more sky brightness) as the Sun altitude decreases (less sky brightness); in order the contrast to remain the same.

4. PROPOSED CRITERION

The criteria discussed so far give successful results for thin crescents to test the earliest visibility. Except Ilyas, they are valid for W < 1'. Ilyas (C) extends it up to 2', where the curve becomes nearly horizontal. This work aims to propose a visibility criterion, which can also be used for thicker crescents including daytime visibility, when the Moon is visible together with the Sun.

We shall investigate the sky darkness change for broader altitude values and consider the atmospheric extinction which influences the Moon illumination.

4.1 "Best Time" of Visibility

The coefficient (slope) of M in the former Hoffman inequality, which equals to 0.31, tells us that for every degree increase in Moon altitude, 0.31 degree decrease in Sun altitude will be necessary to remain the sky darkness the same. This can also be verified on the graph in Figure 3. A horizontal line representing a fixed brightness level crossed by the θ=80° and =85° curves form a gap of about 1.5 degrees of Sun altitude, except for daytime.

The earliest visibility of a waxing crescent occurs just after sunset, whereas the latest visibility of a vaning crescent is observed just before sunrise. During the sunset/sunrise, when the Sun is near the horizon, the Sun and Moon altitudes change in the same rate, such that ARCV remains constant. However, since their contributions to the sky brightness are different (nearly 1:3), the brightness level at the instantaneous Moon altitude decreases as they move down, increasing the contrast and favoring the visibility. Theoretically, the Moon would be best distinguished just before it sets.

But it is well known that after a certain altitude, the Moon starts to fade until it disappears. Bruin's graph visualizes this characteristic (Figure 7). For a crescent width of 0.25', he proposed that the Moon will be just visible at S = -2° (point A), best visible at S = -4° (point C) and it will again vanish at S = -8° (point B). According to Bruin, the point of best visibility appears to be asymmetric nearer to point A. Bruin's work shows that the best visibility point shifts to left as the crescent width increases.

Yallop discovered that these best visibility points are located on a straight line (drawn in red) with a slope of 9/4, i.e. 9 * h = 4 * (h+s). h and s are the Moon and Sun altitudes, respectively, and h + s = ARCV. So he claimed that the "best time" of visibility occurs when 4/9 of the time between sunset and moonset has passed. He therefore asserted the best time for visibility as 4 * h = 5 * s. The recent visibility assessments of Yallop, Odeh, Qureshi and others, which include ARCV as the sky brightness parameter, assume this 4/9 time as the onset of visibility.

Sultan, by making use of a photometric model, demonstrated in 2006, that the best visibility occurs at near M = 2.5° for elongations above 7° (Danjon limit), independent of ARCV. He related this discrepancy to the sight altitude.

4.2 Atmospheric Extinction

We will now explain the effect of extinction, which is the reason why the Moon fades as it approaches the horizon.

Extinction is the decrease of the brightness of any sky object due to the growth of the optical thickness as the zenith angle increases. Figure 8 visualizes the change of optical thickness in kilometers with respect to observation angle in degrees [5]. This increase stems from the spherical shape of the atmosphere. The light rays travel a longer distance through the atmosphere as approaching the horizon. After some point, namely the maximum visibility point, the change of decrease in the object's brightness exceeds the gain in the contrast and the visibility starts to decline.

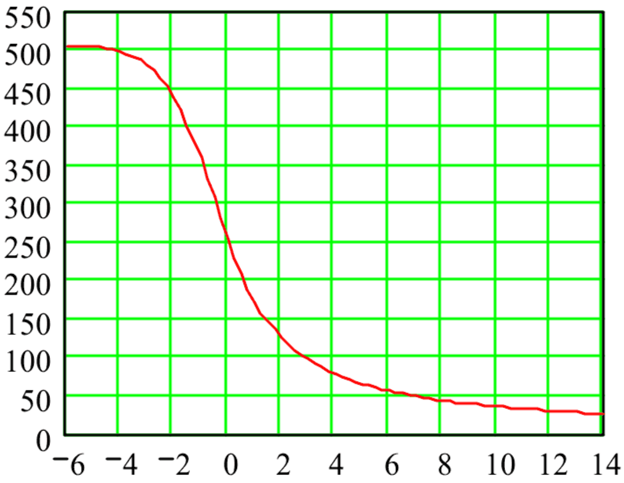

Figure 8 – Optical Path Length vs. Elevation

Ilyas, in 1994, cited that the increased atmospheric extinction when the Moon is closer to the horizon should be compensated by a shift in altitude. Furthermore, he categorized all the existing criteria with two parameters as 4th order, and stated that the 5th order criterion should include extinction [9]. The attenuation to be compensated by an altitude shift can be expressed as F(θ), which is proportional to the air mass (optical path length) in the direction θ [19].

The visibility will deteriorate as the Moon altitude gets closer to zero; so the shift must be negative for decreasing M. Therefore we should incorporate a non-linear shaping-function F(M) instead of the constant-slope 0.31*M, as to compensate for extinction. For very small Moon altitudes in the vicinity of the horizon, the slope should increase heavily, as to follow the optical path growth shown in Figure 8.

The best contrast for visibility occurs when the slope of F(θ) is unity; i.e. visibility is nearly constant as the Moon and the Sun travel down together. The point C of the *h+s* curve in Bruin's graph clearly visualizes this fact; it is the moment of best visibility and the slope of the related *h* curve at that moment is unity.

If the reason for the best visibility phenomenon is the atmospheric attenuation, it should not be related to the crescent width; it is function of the Moon altitude only. Hence the modeling by Sultan seems reasonable. However, the best visibility is neither a function of altitude, at least for the first few kilometers, because the mass density change has a more or less constant slope according to the Standard Atmosphere Model. So we will assume the maximum contrast to appear at M = 2.5° for all conditions.

Thus we will adjust (empirically fit) our shaping-function such that its slope will be unity at the best visibility point, i.e. M =2.5°:

```
F(M) = −0.28 / tan(M + 1.5)
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The slope of the shaping-function is 0.38 at M = 5° and it decreases for increasing M (Figure 9). We will modify the former Hoffman equation such that our visibility criterion conforms to the available observation records and also to the daytime visibility observations, as follows:

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F(M) – S + 6 * √W > 4.9
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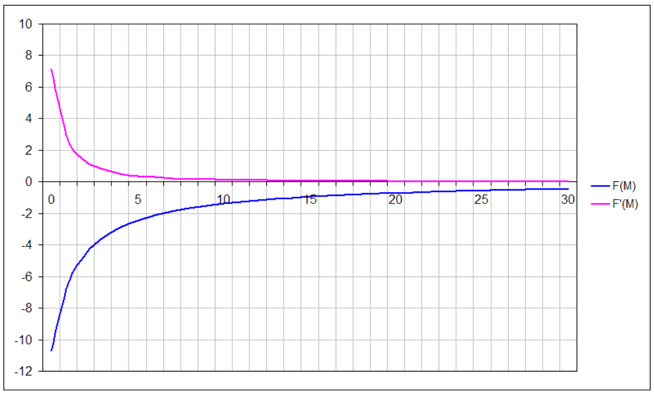

Figure 9 – Shaping-Function F(M) and its Slope

The coefficient of √W is here larger as compared to the Hoffman's findings. This should be caused by the introduction of F(M) which looses slope as W grows (increasing DAZ and decreasing ARCV). This attenuation is now compensated by a larger gain of √W.

4.3 Daytime Visibility

Figure 3 shows that the sky brightness flattens when the Sun altitude becomes positive. It implies that the brightness will remain constant for some value of Sun altitude, which can also be deduced from Figure 8. We will assume this corner angle as 5°, where the optical path nearly doubles. This is also the value taken for the "fading" limit of the Sun (start of *makrouh* timing). So if the Sun altitude is greater than 5°, the sky brightness is almost constant and the crescent visibility (contrast) will be a function of the Moon altitude only, for a fixed crescent width. The visibility will enhance a little for higher Moon altitudes, because the sky brightness diminishes as approaching the zenith. So if we limit the Sun altitude to 5° in our inequality, the daytime visibility will be affected by F(M) only, which mimics this loose dependency of the brightness to the Moon altitude.

Another issue is the crescent width: When the thickness exceeds a critical visual angle and the Moon cusps become resolvable by the eye, the width dependency of the visibility ceases and the necessary contrast will be almost constant. Sultan's model gives the limiting contrast roughly as 0.003 and the corner angle as 5 arc-minutes for daytime sky brightness [20]. So we will limit the crescent width to 5' in our criterion. With the consideration of daytime visibility, the correspondent stipulation we offer will be as follows:

```
-0.28 / tan(M + 1.5) - min(S,5) + 6 * √min(W,5) > 4.9
```

In any case, the crescent will not be visible when its apparent upper limb goes under the horizon. This implies that the topocentric Moon altitude should roughly be greater than -0.5°. The exact value depends on the crescent width as well as the latitude of the observer sight.

4.4 Effect of Height

With increasing height above sea level, air density and optical path length decline, lowering the scattering and diminishing the sky brightness for all zenith angles. Since the Moon illumination is not affected, the contrast will be higher, favoring the visibility. It is a common practice to climb a nearby mountain in order to witness the first emergence of the thinnest crescent. Note that in March 2002 on a sight at 2,200 meter, the crescent with ARCL = 8.6° has been distinguished [14]. Similarly, calculation using a photometric model results in a minimum ARCL = 8.5° for 2,000 m height [7].

The apparent horizon shifts down an angle of E with increasing height above sea level. If S, M and E are small enough, the effect of height can be approximated to a Sun altitude shift equal to E, which should be added to our visibility equation:

```
E = arccos(R / (R + H))
```

Here R denotes the Earth radius and H the height of the observer's sight. This height model tells us that the Danjon limit could be reached at a height of 4,000 m.

4.5 Probability of Visibility

The q-value defined by Yallop to quantify the ease of visibility has a minimum value of 0.216 for "easily visible" and a maximum value of -0.014 for "optical aid to find the crescent" [6]. The difference is 0.23 which corresponds to 2.3 degrees. This spread reflects the uncertainty related to the observations and incorporates atmospheric conditions as well as observers' capabilities. Ilyas [9] and Schaefer [13] also investigated this effect and found a similar spread of ±1°. In the Hoffman's equation, this spread represented with the coefficient ± σh_S / (1+y), which is calculated as ± 0.99°. Therefore we will take the spread as 2°.

Accordingly, we incorporate the percent probability (P) into our criteria and the final criterion becomes:

```
-0.28/tan(M+1.5)-min(S,5)+6*√min(W,5)+arccos(R/(R+H))-P/50 > 3.9
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The resulting visibility algorithm is sketched below in Figure 10:

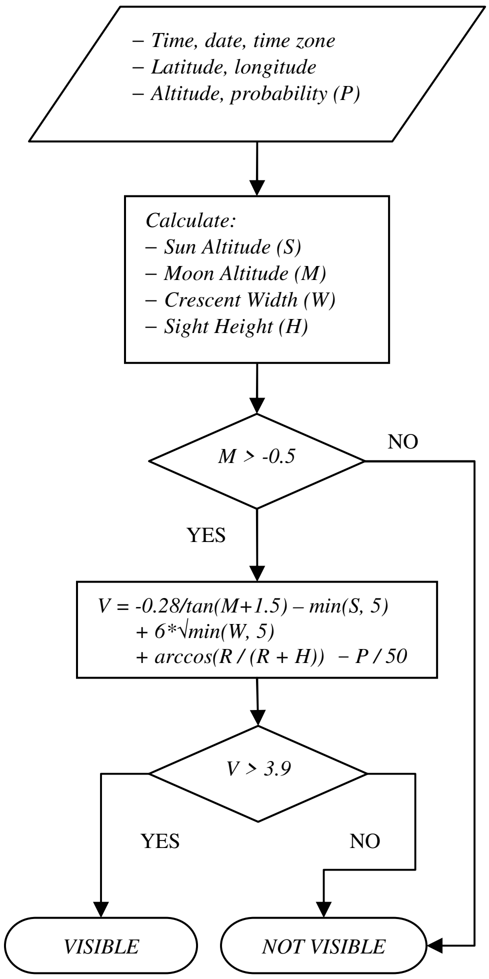

Figure 10 – Flowchart of Proposed Visibility Algorithm

The algorithm claimed can be plotted on an altitude graph, as to compare to Bruin's analysis. Figure 11 and 12 are the h and h+s graphs, respectively, for a bunch of W values. It can be marked in Figure 11 that the slope of each curve is unity at h = 3.5° (M = 2.5°). The dotted line in Figure 12 is the corresponding h = 3.5° condition, where the visibility is maximum.

The curves are extended up to s = -5°, namely 5 degrees above horizon, to display daytime visibility. A 3.5' thick Moon appears to be visible during daytime if h > 11°, whereas a 5' (and thicker!) Moon will be visible after h = 4°. The curves are sketched for sea-level at 50% probability; the visibility limit of a 2' crescent at s = 0 (a.s.l.) may spread between altitudes 3° and 5.5°; or a gibbous can be observed (P = 50%) when h = 3° (M = 2°) at 1000 m above sea level.

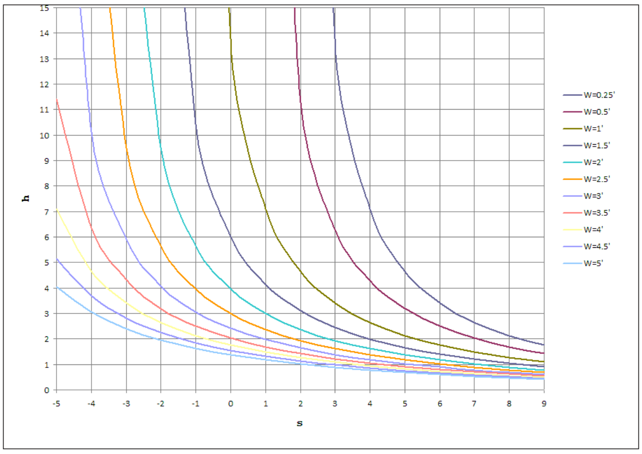

Figure 11 – h versus s Graph of Proposed Criterion

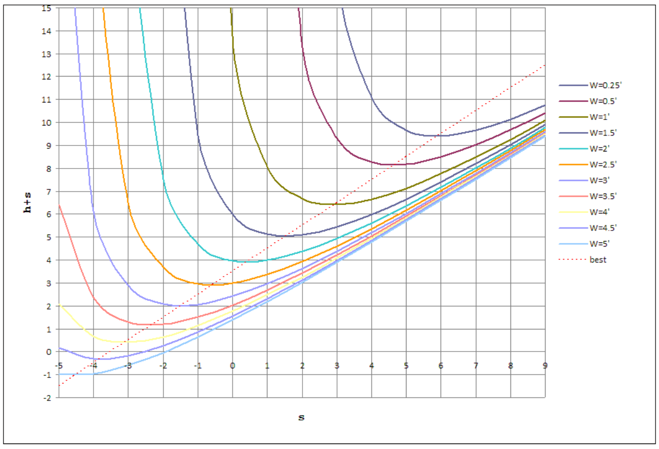

Figure 12 – h+s versus s Graph of Proposed Criterion

Those curves are consistent both with Yallop's criterion and Sultan's findings. In fact, Sultan's photometric calculations show the theoretical best visibility point, where the Moon is visible for only an infinitesimal duration. On the contrary, the real naked-eye observations, as analyzed by Yallop, necessitate the Moon to stay visible for a sufficient amount, say 10 minutes, so that the observer scanning the sky surface can catch and definitely identify the hardly distinguishable crescent. The best visibility sights with low DAZ / high ARCV values are generally located at low to moderate latitude, where the Sun and Moon follow a steep trajectory. They move down approx. 2° in 10 minutes. So it will be necessary that the Moon is already visible at 2.5° + 2° = 4.5°. The 4/9 rule gives nearly the same topocentric altitude for small DAZ values (see Test Case #1). For a sight at higher (say 65°) latitude, where ARCV is smaller (say 8°) due to the slant trajectory, the Sun/Moon will move only 1° in 10 minutes, such that the Moon must now be visible at 3.5° (see the Test Cases #2 & #3). Thus the 4/9 rule can be meaningful for determining the onset point of visibility for observations of crescents with W < 1'. 4/9 rule will fail for thick crescents, as the Moon altitude (*h*) at best visibility will approach to zero, which is not realistic because of atmospheric extinction.

We additionally prepared an extended ARCV/DAZ graph in Figure 13 as to compare with Ilyas (C) criterion. The curve beyond DAZ = 30° is no more horizontal here, but decreasing with a lesser slope.

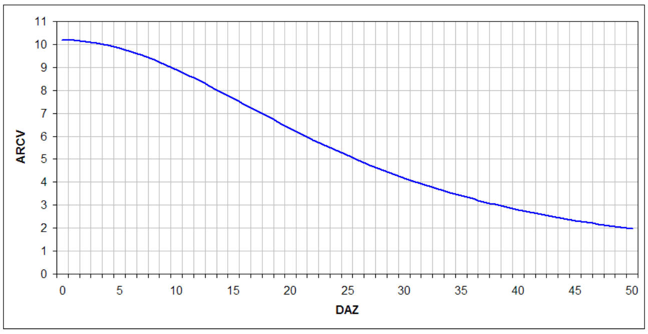

Figure 13 – ARCV versus DAZ Graph of Proposed Criterion

5. DEMONSTRATION TOOL

To demonstrate the performance of the criterion and compare its results with the other criteria in literature, a tiny software program has been developed as a screen saver. EHILLE, this screen saver, can be easily configured to supply the necessary parameters (Figure 14). The height can be 10 km at most, useful for airplane visibility simulation.

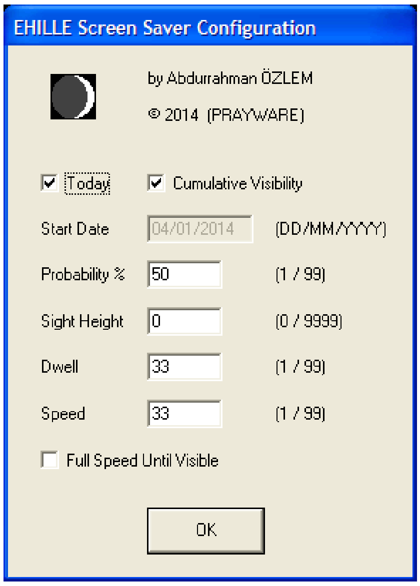

Figure 14 – Configuration Screen of EHILLE

The area of instantaneous visibility is painted on a Mercator map in real-time and the painted areas are then combined to form the cumulative area of visibility, which has the shape of a parabola. The vertex of this parabola represents the "best place" on Earth and the area widens westward, being symmetric on a roughly horizontal line. The position of the vertex is unique for each lunation.

The software first computes the time of conjunction and shifts the Mercator map accordingly, such that the parabola lies more or less on the same place, its vertex being placed near the right border. Next is calculated the start time, which is nearly 3 hours before the first global visibility. Then the time is progressed with the entered speed and the visibility is checked continually.

A relatively simple approach to draw the parabola would be to compute the visibility for each pixel (corresponding to a certain latitude & longitude) by executing the novel algorithm for every minute of time to be tested. However, this necessitates more than 500 million loops for a complete run, requiring a huge amount of calculation time. Therefore the software uses a smart search & track method which speeds up the process nearly 5,000-fold, as detailed below:

Following the start, the software searches the best place on the map for visibility. Beginning from the center of the right border, a vertical search (up and down) is performed as to maximize ARCV to find the latitude where the Moon is vertical to the Sun (DAZ = 0). Consecutively, a horizontal search determines the longitude where the Moon altitude is 2.5°, which is most favorable condition according to our algorithm. The combined search fixes the position with the highest possibility of visibility, and this position is tested & updated only once for each time using the algorithm. The time is then incremented one minute and the search is repeated.

After detection of the maiden visibility at this best place, its coordinates and the local time are displayed on the screen. Now, the software checks the visibility in a vertical scan and saves the latitude limits of visibility, up and down, which forms the border points of the parabola. The connection line is then painted. This scan is repeated for neighbor longitudes, left and right, until visibility ceases. The instantaneous visibility area is formed thereby. For the next minute, the software shifts the former "best place" to left (¼°) and the horizontal scan is initiated from the saved points, considerably shrinking the computing time.

A screenshot of the tool is displayed in Figure 15 to visualize the output. The graph looks very similar to those obtained through the famous Moon Calculator program by Dr. Monzur Ahmed.

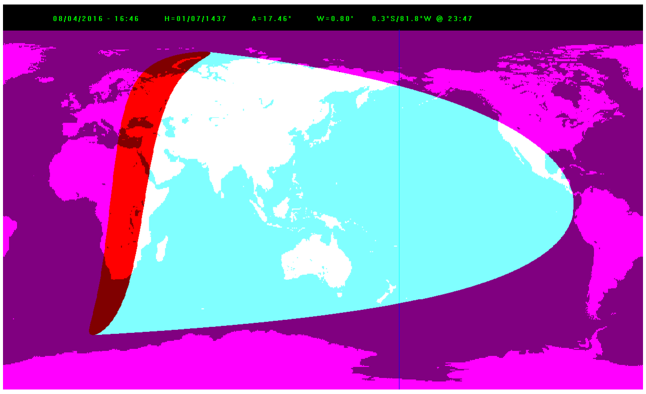

Figure 15 – Visibility Parabola Obtained by EHILLE

6. COMPARISON & CONCLUSION

6.1 Performance Evaluation

Several cases will be analyzed in this section and compared with the criteria available in literature, as to verify the validity of the proposed method. The performance is to be evaluated by checking whether the error calculated for 50% probability remains within the ±1° spread. Since our criteria uses topocentric Moon elevation, we will consider the parallax of Moon (≈ 1°) in our calculations.

The first four cases test the algorithm for crescent widths of 0.25' ~ 1'. ARCV and DAZ values are obtained by the 3rd degree polynomial offered by Yallop. The next three cases use the Ilyas (C) criterion (Figure 6), suitable for larger crescent widths. Except the last case, the Moon altitudes (M) have been selected by regarding the 4/9 rule, namely M = ARCV x 5/9 - 1.

<u>Case #1</u>: W = 0.25' (ARCV = 10.30°, DAZ = 0.29°)

Regarding the 4/9 rule, the Moon altitude will be taken as 4.72°. The necessary Sun depression is calculated as 4.47°. This gives an ARCV= 4.72°+4.47°+1° = 10.19°, that is 0.11° lower than the given value of 10.30. Hence the result has an error of -0.11°.

<u>Case #2</u>: W = 0.5' (ARCV = 8.85°, DAZ = 11.65°)

The Moon altitude will be taken as 3.91°, using the 4/9 rule. The necessary Sun depression is calculated as 3.61°. The result has an error of -0.32°.

<u>Case #3</u>: W = 0.75' (ARCV = 7.46°, DAZ = 16.31°)

The Moon altitude will be selected as 3.15°. The necessary Sun depression is calculated as 3.15°. The result has an error of -0.17°.

Case #4: W = 1’ (ARCV = 6.15°, DAZ = 19.80°)

The Moon altitude will be selected as 2.41°. The necessary Sun depression is calculated as 2.99°. The result has an error of 0.26°.

Case #5: ARCV = 5.5°, DAZ = 23°

The Moon altitude will be taken as 2.06°. The crescent width is 1.30’ and the necessary Sun depression is calculated as 2.57°. The error is 0.13°.

Case #6: ARCV = 5°, DAZ = 25°

The Moon altitude will be taken as 1.78°. The crescent width is 1.51’ and the necessary Sun depression is calculated as 2.43°. The error is 0.20°.

Case #7: ARCV = 4.5°, DAZ = 28°

The Moon altitude will be taken as 1.5°. The crescent width is 1.86’ and the necessary Sun depression is calculated as 2.07°. The error is 0.07°.

Case #8:

Here we test the validity of the height compensation. For 2,200 m height at a Moon altitude of 3.72°, our criterion gives the minimum ARCL = 8.66°. The error is 0.06°.

Case #9:

This is the 2nd height compensation validation, now against the photometric model. We take the Moon altitude as 2.5°. For 2,000 m height, our criterion gives the minimum ARCL = 8.5° exactly, without any deviation.

Despite the simplicity of the offered model, the obtained results seem well within the uncertainty spread:

- Comparison with the Bruin’s graph exhibits a tight match up to W = 1’. Bruin’s work is regarded as consistent only for low latitudes (small DAZ and W). From mid latitudes onwards, great discrepancies are found in the results [10][15]. The graphs obtained by our work should be more accurate for a wider range of parameters.
- Verification against Yallop’s criterion, based on a large observational dataset, also comes out to be successful for W < 1’.
- Test with Ilyas (C) graph shows high consistency up to W = 2’. The flat portion beyond 2 arc-minutes of his graph is regarded as “overestimated” [10], so the curve produced according to our criterion seems to be more realistic.
- The sight altitude compensation fits with the theoretical modeling and practical observations. Beyond 10 km however, it would show considerable deviation.
- Limiting the Sun altitude at 5° and the crescent width at 5’ for daytime visibility results in sharp corners at those values. The real transition should rather be smooth. Nevertheless, several daytime observations performed at these conditions showed that the maximum deviation is within the uncertainty spread (typically less than 1°) and therefore it would not be worth to insert complicated correction terms.

6.2 Discussion on Islamic Calendar

When relying on local calendar, the Visibility Separator Parabola [3] clearly defines the new lunar month; the crescent is visible on the area within the parabola and the next month begins, whereas inhabitants on the area outside the parabola should wait for another day. However, if we consider a global calendar instead, the problem is how to set the International Lunar Date Line (ILDL), or better “Lunar Month Line”. An old but rational rule is test whether the crescent will be visible on the Earth before 23:59 (UTC), i.e. midnight at Greenwich. If yes, the following day is declared as the first day of the new Islamic month; if not, it will be the last day of the old month. This rule is regarded as reasonable because at that time Sun sets at the 90°W meridian and the crescent can only be seen west of that, where the Great Ocean resides. So if the crescent is observable after 23:59 (UTC), probably nobody will be able to see the crescent on that night; otherwise, at least some people on the west coasts of the American continent may see the crescent, and, since meanwhile the night still continues on a large majority of the Earth, that night and the following day can be declared as the beginning of the new lunar month. Figure 15 visualizes this case, where the visibility starts at 23:47 (UTC), just before the midnight at Greenwich and the crescent is only visible in the far west of America.

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